The Optimal Inflation in a Backward-Induction Game

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Abstract: Within the context of a two-stage sequential game that is played among three players: workers, central bank, and firms, we try to look for an optimal inflation rule to be implemented by the central bank in order to maximize social welfare. In stage one, workers choose a wage at which they are willing to work; in the second stage the central bank chooses the rate of inflation after seeing the wage for which people have contracted for, and then firms decide how many workers they will hire depending on the real wage. By following this strategy, it is determined that the central bank's optimal inflation rule is greater than its target inflation rate unless the target employment level (in long terms) is zero. In a different scenario at which firms move first, the inflation rate will equal the targeted level.

Keywords*: optimal inflation rate; expectations; monetary policy*
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Introduction

Some economists believe that economic growth can be enhanced and price stability achieved by implementing a monetary policy rule. Rules are argued to reduce policy mistakes, improve the transparency of policy, and end political influence on policymaking. Rules have been suggested for Latin American economies that for many years have struggled with high inflation, unemployment, and financial crises (Zarazaga, 1995). In fact, a strong consensus prevails among economists that discretionary monetary policy generates economic instability, is biased toward inflation, and is open to

special interest politics. Friedman (1968) argues that policy efforts to peg low interest rates or high employment only produce higher interest rates and inflation later. Monetary authorities can control money growth, however, and it is changes in money that affect economic stability. Furthermore, rule advocates believe that monetary policy strategy and targets frequently shift, producing a time-inconsistent and unpredictable policy. The result is a substantial decline in people's faith that the central bank is committed to low inflation.

Complicating this problem are the rational expectations of economic agents who either must be fooled into expanding economic activity or who will react before any policy is implemented in a way that makes activist policy ineffective in changing employment and output (Sargent and Wallace, 1975). Expectations formed from experience lead people to believe that activist monetary policy will contribute to economic instability and excessive inflation (Kydland and Prescott, 1977). Therefore, a rule that commits the monetary authority to a nonactivist, nondiscretionary policy makes the economy more stable by creating a more credible, more certain policy aimed at price stability (Barro and Gordon, 1983a). In contrast, discretion leads to an inflationary bias in monetary policy when policymakers err towards achieving unemployment levels below the natural rate.

Theoretically, a credible noninflationary policy stance could costlessly reduce inflation and achieve lower unemployment compared to a discretionary policy (Barro and Gordon, 1983b). A monetary rule prevents policy from being manipulated by government fiscal and political interests. Government's attempts to obtain resources through printing money create inflation, which taxes the public and reduces the real value of property, all without legislation. A price stability mandate, achieved through a rule, protects private property and keeps state finance honest. Rules also prevent political pressures that distort markets or incentives, and limit the government's ability to placate special interests (Goodfriend, 1997; Cukierman, 1992).

Moreover, there are some influential studies that have examined the design of an optimal monetary policy of a central bank that tries to keep both low inflation and low unemployment as playing a sequential game. Fudenberg and Levine (1992) studies inflation rule issue in sequential-move stage games with a single long-run player whose choice of stage-strategy is imperfectly observed by short-run opponents.

Skott (1997) considers a unionized economy where individual unions recognize the existence of a link between their wage demands and the average rate of inflation, and they act as Stackelberg leader's vis-à-vis the monetary authorities. There are three players: a central bank maximizing its objective function, wage setting unions that anticipate the reaction of the central bank, and a government that sets the central bank's objective function. This paper studies the optimal specification of these central-bank objectives.

In the presence of labor unions, the monetary policy game can lead to radically different results: a central bank that is completely indifferent to the level of inflation may obtain outcomes with high employment rates and zero inflation while inflation-averse central banks generate stagflation with positive inflation and low rates of employment. Yuan, Miller and Chen (2006) show how to develop a general method for making optimal and consistent policy in simple models. They also have three players: the wage setter, the central bank and the firm. In their model, the wage setter and the firm sign a wage contract, where the wage setter sets the nominal wage, and the firm sets the amount of labor that it would like to hire. The central bank will then implement its policy decision to minimize the loss function.

Motivated by the important role that major market participants play on this subject, our paper adds to the literature by applying a game theory technique via a sequential game that is played among workers, firms, and the central bank for an optimal monetary policy to be implemented by the central bank in order to maximize the social welfare.

The paper is structured as follows. In Section 2, we present the model and derive the pay off function and loss function. Then we analyze the first scenario when firms move last in the three-player game, and establish the main results concerning the central bank's inflation rule in Section 3. Section 4 examines a different scenario in which firms move first. Section 5 concludes the paper.

The Model

In our model, there are three players, the central bank, firms, and workers (households). The central bank cares about hitting specific inflation and unemployment targets. Its pay off function is given by

$$V_c \ (w, \ L, \ \pi) = - (L - \hat{L}) \ 2 - \mu \ (\pi - \hat{\pi})^2 \tag{1}$$

where π equals the natural logarithm of the price level, that is $\pi = \ln(P)$. $\hat{\pi}$ equals the targeted inflation rate, L denotes the natural logarithm of the employment level, w is the nominal wage rate, and μ is the weight that the central bank places on the inflation target relative to an employment target (a trade-off parameter). The first term of the loss function penalizes the central banker for missing the employment target while the second term penalizes it for missing the inflation target. The central banker controls a monetary instrument, which enables him to select the rate of inflation in each period.

Here we will assume that the inflation rate is determined uniquely by the central bank policy instrument, which is the rate of growth of the money supply (m). That is

$$\pi = \Psi(m) \tag{2}$$

where m = ln (M_t) – ln (M_{t-1}) and M is the nominal money supply.

Workers value both income and leisure. We will say that the aggregate amount of labor supplied to the market is an increasing function of the real wage rate (r). This means that workers contract for a nominal wage in the first period, but then adjust the amount of labor they supply to the market upon seeing the market clearing real wage in period two. Specifically,

$$L^S \ (r_t) = \ \alpha r_t, \quad \alpha > 0 \tag{3}$$

where the real wage r_t = w – π, and α is the elasticity of labor supply ($\alpha > 0$). A 1% decline in the real wage reduces the amount of labor supplied to the market by α%. Workers are concerned about unemployment. Their payoff in the game is then

$$V_W(\ w\ ,\ L\ ,\ \pi\ ) \ = \ - [L - \alpha \ (w - \pi)]^2 \tag{4}$$

where L is the number of people that have jobs, and α (w – π) is the number of people that desire to have jobs. The difference is unemployment. The minus sign out front insures that the payoff to workers goes down as unemployment rises. In order to make the payoff as large as possible, the workers should work for a wage that makes unemployment as small as possible.

Firms want to maximize profits. In our model, their only factor of production is labor. Their aggregate demand for labor is given by

$$L^d \ (r_t) \ = -\lambda \, r_t = \lambda \ (\pi - w), \tag{5}$$

where L^d is the quantity of labor that firms would like to hire, and λ is the elasticity of labor demand. Therefore, firms' payoff is determined by the difference between what they do hire, L, and the number of workers they wanted to hire; their loss function will be

$$V_F \ (w, L, \pi) = -\left[L - \lambda(\pi - w)\right]^2 \tag{6}$$

Note that the three payoff functions have the same form at which they are quadratic in essential arguments and a minus sign in front of each of them. This assumes that each player has a squared-error loss function. The minus sign turn each of the loss minimization problems into a maximization problem.

As mentioned above, the purpose of this paper is to look for an optimal monetary policy to be implemented by the central bank in order to maximize the social welfare. The optimal policy plan is retrieved via a sequential game that is played once over two periods. The workers choose a money wage at which they are willing to work in period one and in the second period, the central bank chooses the rate of inflation after seeing the wage for which people have contracted and then firms decide how many workers they will hire based on the real wage.

In a game theory model, especially a microeconomic model with a social planner (e.g., the central bank), we define the optimal policy as the social planner's ex ante plan (if implemented) that produces a Pareto efficient outcome according to some social welfare criterion.

First, note that the labor market clears (in equilibrium) when the quantity of labor supplied equals the quantity of labor demanded ($L^s = L^d$). Then we have

$$\alpha r_t = -\lambda r_t \tag{7}$$

where α (w – π) is the supply of labor, λ (π – w) is the demand of labor, and w – π is the real wage rate.

This equality can hold only when the logarithm of the real wage rate is zero. That is, equilibrium occurs when the nominal wage divided by the rate of inflation is one. If the log of the real wage is zero, then the log of either labor supply or labor demand must also be zero. Hence, as a result of our normalization rule and equilibrium in the labor market, the natural level of employment is one.

First Scenario

We will apply backward induction to find the equilibrium solution to the game. The three players move as prescribed above and as shown in Table 1.

Table 1

	Workers	**Central Bank**	**Firms**
Preferences	$-\left[L - \alpha\,(w - \pi)\right]^2$	$-(L - \hat{L})\,2 - \mu\,(\pi - \hat{\pi})^2$	$-\left[L - \lambda(\pi - w)\right]^2$
Strategies	w	π	L

According to backward induction, since firms move last, we begin with them. Firms choose the only variable over which they have control (L), so as to maximize V_F. That is, firms want to minimize the difference between their actual employment and the profit maximizing level of employment. Thus, we find the first derivative and set it equal to zero, then solve for the optimal level of employment given the earlier moves by workers and the central bank.

$$\frac{\partial\,\{-\,[L - \lambda\,(\pi - w)]\}^2}{\partial L} = -2\,[L - \lambda\,(\pi - w)] = 0 \qquad (8)$$

$$\Rightarrow\ L^*\,(w,\pi) = \lambda\,(\pi - w)$$

The central bank moves next to last and chooses its inflation rule so as to minimize the loss from missing its employment and inflation targets. After substituting in the firms' choice of employment, the central bank wants to choose π so as to minimize

$$V_C\,(w, L^*\,(w, \pi), \pi) = [\lambda\,(\pi - w) - L]^2 + \mu\,(\pi - \hat{\pi})^2 \qquad (9)$$

Thus, we find the first derivative and set it equal to zero, expand the multiplications, and solve for the bank's inflation rule. The rule depends on the wage chosen by workers in the first period, the target inflation rate and the target level of employment.

$$\frac{\partial\{[\lambda(\pi - w) - \hat{L}] + \mu(\pi - \hat{\pi})^2\}}{\partial \pi} = 2[\lambda(\pi - w) - \hat{L}]\lambda + 2\mu(\pi - \hat{\pi}) = 0 \qquad (10)$$

$$\Rightarrow \pi^* = \frac{\lambda w + \lambda \hat{L} + \mu \hat{\pi}}{\lambda^2 + \mu}$$

Note that the bank's inflation rule will not be equal to their target inflation rate, unless the elasticity of demand for labor turns out to be zero ($\lambda = 0$).

We now turn to the first movers. Workers must make their move, that is accept a job at a particular wage before the bank's inflation rule or firms' employment rules are revealed to them. In this economy workers must form some expectation about inflation. We assume that workers can look ahead and use the central bank's optimal inflation rule as the basis for their own expectations.

Specifically, they believe that

$$\pi^e = \pi^*\,(w) \qquad (11)$$

Substituting this expression for inflation expectations and the firms' employment rule into the payoff function for workers, we get

$$V_W (w, L, \pi) = - [L^* (w, \pi^*) - \alpha (w - \pi^*)]^2 \tag{12}$$

Making the substitution for labor demand and multiplying out the minus sign on the term for those seeking to work, we get

$$V_W (w, L, \pi) = - [\lambda (\pi^* - w) - \alpha (w - \pi*)]^2 \tag{13}$$
$$\Rightarrow V_W (w, L, \pi) = -[(\lambda + \alpha)(\pi^* - w)]^2$$

To find the *w* that maximizes this payoff we will differentiate with respect to *w*. Setting the result equal to zero and solving for w provides

$$\frac{\partial\{-[(\lambda+\alpha)(\pi^*-w)]^2\}}{\partial w} = -2(\lambda + \alpha)^2 (\pi^* - w)(-1) = 0 \tag{14}$$

Rearranging the above equation, we can see that workers will agree to work for a wage equal to the central bank's inflation rule.

$$w^* = \pi^* \tag{15}$$

Since the rate of inflation and the wage rate are the same, the real wage (in log terms) will be zero, and we conclude that the labor market is in equilibrium.

Substituting this result in the expression of the central bank inflation rule, from equation (15) we can get

$$w^* = \frac{\mu\pi + \lambda\hat{\pi} + \lambda^2}{\mu + \lambda^2} \tag{16}$$

Collecting on *w* and simplifying this equation, we find

$$w^* = \hat{\pi} + \frac{\lambda}{\mu}\hat{L} \tag{17}$$

Now that we know the nominal wage chosen by workers, we can substitute it back into the expression for the central bank's inflation rule to get

$$\pi^* = \hat{\pi} + \frac{\lambda}{\mu}\hat{L} \tag{18}$$

Notice that the central bank's inflation rule turns out to be greater than its target inflation rate unless the target employment level (in log terms) is zero. Target employment in logs is zero when the actual target is 1, or the target equals the "natural" level of employment. The natural level of

employment corresponds to that competitive wage at which all those seeking employment find it.

Different Scenario

We can also check another scenario at which firms move first, but the central bank and workers keep the preferences and strategies as before.

Using the same technique - backward induction, since workers move last, the game begins with them as they choose w to maximize V_W. In the previous section, we have got $V_W = -[L - \alpha(w - \pi)]^2$.

$$\frac{\partial(V_W)}{\partial w} = -2\alpha[L - \alpha(w - \pi)] = 0 \tag{19}$$
$$\Rightarrow \quad w^* = \pi + (1/\alpha)L$$

Now comes the central bank moves next and it chooses optimal π to maximize V_C, which equals $-(L - \hat{L})2 - \mu(\pi - \hat{\pi})^2$. Then we can get

$$\frac{\partial(V_{WC})}{\partial \pi} = -\mu(\pi - \hat{\pi}) = 0 \tag{20}$$
$$\Rightarrow \pi^* = \hat{\pi}$$

In this case, this is different than the optimal inflation rate, which equals the targeted inflation rate. This is because that the central bank's preference function parameters are all unknown at that stage, and it does not replace any parameters by their values) i.e. in the first scenario, L was replaced by its value from the firms' maximization problem.

The central bank does not promote any surprising inflation within the second scenario. As a result, its moves are anticipated by both workers and firms (notably if the targeted inflation is declared), and thus no firm will expand output and employment. However, if the central bank sticks to this rule, it will gain credibility, which is helpful in maintaining low inflation rates through agents' expectations.

For the first player, firms, they would choose L to maximize V_F. In section 3, we have $V_F = -[L - \lambda(\pi - w)]^2$.

Both π and w are now known, so we can substitute their values in V_F, and we rewrite the above equation to receive

$$V_F = -[L - \lambda(\pi^* - (\pi^* + (1/\alpha)L))]^2 \tag{21}$$
$$\Rightarrow V_F = -[L - \lambda\pi^* + \lambda\pi^* - (\lambda/\alpha)L]^2$$
$$\text{Then} \quad \frac{\partial V_F}{\partial L} = -2\frac{\lambda}{\alpha}[L - \lambda\pi^* + \lambda\pi^* - (\lambda/\alpha)L]^2 = 0$$
$$\Rightarrow L^*\left(1 - \frac{\lambda}{\alpha}\right) = 0$$
$$\Rightarrow L^* = 0$$

Here, L^* is zero, which means that employment level is one (full employment). This gives us a hint that inflation does not matter in that kind of model in determining employment levels, since with different inflation rates in both scenarios, we got the same full employment level.

Moreover, if substituting L in the wage equation, the result will become w = π, which is also the same result as we got in the first scenario. Then the real wage is always tight with inflation. This ensures that in backward induction three-player game, sequences of playing neither affects the level of neither employment nor wage inflation equality. However, it does affect the value of inflation (and wage rates), which are lower in the second scenario than the first. This, in fact, is a result of changing the sequence of the game.

Conclusion

This paper considers two scenarios of a backward-induction game, at which workers move first and firms last in the first scenario, while in the second scenario, firms are the first movers. In the first scenario, as workers care about income and leisure, they must forecast the rate of inflation when they enter into a wage contract. If the rate of inflation turns out to be different from the forecasted rate, then workers will supply either more or less labor than they had planned. We find that workers always tie the level of wages with the inflation rate.

Firms care about profits, and they will increase output and employment only if they think that their prices have risen relative to other prices. The central bank cares about hitting inflation and employment targets. Since the model allows a trade-off between those two variables, the central bank minimizes its loss function and gets the optimal inflation target. The first scenario shows that the central bank's optimal inflation rule is greater than its target inflation rate unless the target employment level (in log terms) is zero. Furthermore, the results show that under such circumstances, the labor market clears at the full employment level.

In a different scenario, things stay almost the same: full employment and a tie between the wage level and inflation. However, inflation tends to be equal to the targeted level which is now lower than the one obtained under the first scenario. In our future studies, we would like examine situations in which workers, the central bank, and firms have preferences that are different from the present paper.

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